Volterra/Hybrid Equalization of Nonlinear ISI in a Magneto-Optic Data Storage Channel

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Abstract: - This paper presents performance analysis of an adaptive Volterra/Hybrid equalizer in a nonlinear magneto-optic data storage channel. In a data storage system, nonlinear intersymbol interference (ISI) is one of the limiting factors for high storage density. Volterra and Hybrid (combination of linear and Volterra) equalizers are proposed and developed for compensation of nonlinear ISI. The use of Volterra/Hybrid equalizers provided an extra 1.3-1.5 dB SNR gain over linear equalizers. Performance and complexity of these equalizers are compared. Convergence issues are also investigated by examining the eigenvalue spread of the covariance matrix. The techniques proposed here are applicable to other communication systems (e.g. wireless, optical) exhibiting nonlinear ISI characteristics.

Key-Words: - ISI, Nonlinear Equalization, Volterra, Optical Data Storage, SNR, eigenvalue, convergence ratio

1 Introduction

In data storage system the bit sequence is encoded and then marks (symbols) of variable length are generated which are then recorded on the storage medium.

Magneto-Optic data storage is a rewritable storage technology that uses a combination of magnetic and optical methods. Data is written on an M-O disk by both a laser and a magnet. The laser heats the mark to a certain temperature (Curie temperature) at which point the molecules are realigned when subjected to a magnetic field. A magnet then changes the mark's polarity. Reading is accomplished by shining a low power polarized laser beam whose plane of polarization changes depending upon the direction of magnetization.

ISI is defined as the mark (symbol) size deviation between the final readback mark and the original one. This deviation is a function of the mark size itself and its neighboring marks. The readback marks were obtained by recording and reading various data patterns on a magneto-optic drive.

To correct ISI, equalizer is used. Traditionally linear equalizers are used. An adaptive equalizer is used to compensate for channel variations by dynamically adjusting the weights. Linear equalizers were proposed in [6] and RAM (random access memory) based lookup table approach for equalization of nonlinear storage channel was examined in [7]. DFE (decision feedback equalizer) and ADFE (adaptive decision feedback equalizer) are variations of linear equalizer which are implemented using feedforward and feedback linear filters. In the feedback filter decoded symbols are used in the equalization process. In the present work Volterra series which generate nonlinear terms are investigated for compensating the ISI. Modification of Volterra in the form Hybrid equalizer is explored and its performance compared.

The remainder of the paper is organized as follows. Section 2 describes the Volterra equalizer and its mathematical formulations. Section 3 presents the Hybrid equalizer which is a combination of traditional linear and Volterra equalizer. Section 4 covers the convergence analysis of the equalizers. Section 5 develops the SNR expression which is then used as a quantitative figure of merit for performance comparison and finally Section 6 summarizes the main conclusions of this work and its application to other systems.

2 Volterra Equalizer

Volterra series has been successfully used in modeling non-linear systems such as Satellite Communication, Machine tools, etc.

Volterra equalizer utilizes an (N-1)-stage tap delay line, just as a linear equalizer. The difference is that Volterra nonlinearly combines received marks which are then used in the equalization process. Volterra series is represented by the following expression.

$$Z_n = \sum_{n1} w_{n1} Y_{n-n1} + \sum_{n1}\sum_{n2}\sum_{n3} w_{n1} w_{n2} w_{n3} Y_{n-n1} Y_{n-n2} Y_{n-n3} \quad (1)$$
$$+ \sum_{n1}\sum_{n2}\sum_{n3}\sum_{n4}\sum_{n5} w_{n1} \cdots Y_{n-n1} Y_{n-n2} \cdots$$

where Z_n = Output symbols, Y_n = Input symbols and w_n = weights of the equalizer.

The number of terms L is related to the order and number of taps N of the filter as –

$$L = \sum_{i=1}^{(order+1)/2} N^{2i-1} \quad (2)$$

The schematic diagram of a 3-tap 3rd order Volterra equalizer is shown in Fig. 1. It is a modification of a 3-tap linear equalizer. There is another intermediate stage where all the nonlinear terms of the Volterra series are terms generated.

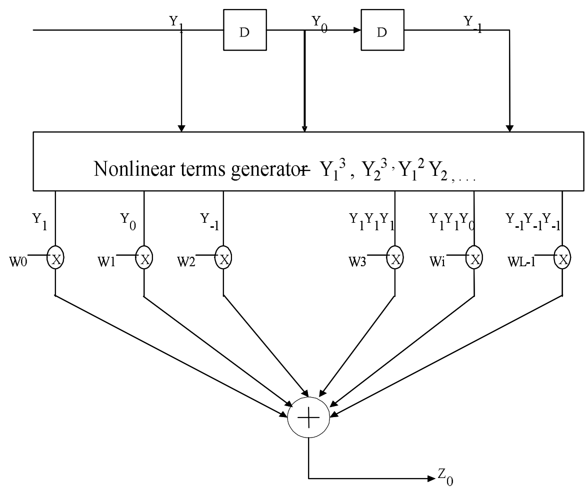

Fig. 1. Schematic diagram of 3-tap 3rd order Volterra Equalizer.

One of the drawbacks of Volterra equalizer is that the number of nonlinear terms generated tends to explode as one goes to higher orders of the series.

Table 1 lists the number of terms generated in the series corresponding to number of taps and the Volterra order used.

Table 1. # of taps, Volterra order and # of terms

# of taps & Volterra order	# of terms
3-tap 3rd order	30
5-tap 3rd order	130
7-tap 3rd order	350
3-tap 5th order	273
3-tap 7th order	2460
5-tap 5th order	3255

3 Hybrid Equalizer

To overcome the issue of large number of terms, Hybrid equalizers are proposed which is a combination of linear and short-tap small-order Volterra filter in the middle of the tap delay line. It is based on the assumption that most of the nonlinearity is because of the most adjacent marks to the mark under consideration. Hybrid equalizer considered consisted of a 3-tap 3rd order Volterra filter in the middle and a large N-tap (N=7, 9, 11) linear filter. The short Volterra in the middle compensates for most of the nonlinear ISI. A further reduction in the number of nonlinear terms can be obtained by discarding the terms with very small relative weights.

In the following sections performance of this hybrid configuration is evaluated and compared with linear and pure Volterra equalizers.

4 Convergence Analysis

Since the nonlinear combining occurs before the tap weights of the equalizer, the outputs of the nonlinear term generator may be considered as inputs to a linear filter. The weight determination is performed using the traditional LMS (Least Mean Square) algorithm as shown below.

$$W(k+1) = W(k) + \mu e(k) U \quad (3)$$

where W=weight vector, U=output vector of nonlinear terms generator (from Fig. 1), e=error, k=k-th update time and μ=step size.

The output of the equalizer is given by –

$$Z = W^T U \quad (4)$$

where Z=output of the equalizer.

To determine the convergence properties, the eigenvalues of the covariance matrix were examined. The eigenvalue spread is defined as –

$\rho = \frac{\lambda_{max}}{\lambda_{min}}$, where λ_{max} and λ_{min} are the largest and smallest eigenvalues of the covariance matrix ($RW_{opt} = P$ where R=covariance matrix, W_{opt} =optimum weights and P=cross-covariance matrix).

The convergence ration ω is given as -

$$\omega = \frac{(\rho - 1)^2}{(\rho + 1)^2} \qquad (5)$$

Intuitively, eigenvalue spread is important as it gives a quantitative measure of degree of correlation of data. Larger the spread, slower the convergence of the algorithm.

Table 2 shows the eigenvalue spread and convergence ratios for various equalizer configurations. From the table one can see that the convergence ratio increases going from linear equalizer to the Volterra equalizer. The convergence ratio of the hybrid equalizer lies in between the linear and pure Volterra equalizer.

Table 2. Equalizer Configuration and Values of Eigenvalue Spread & Convergence Ratio

Equalizer Configuration	Eigenvalue spread	Convergence ratio
5-tap Linear EQ	22.36	0.836
3-tap 3rd order Volterra EQ	29.43	0.873
5-tap 3rd order Volterra EQ	33.45	0.887
7-tap 3rd order Volterra EQ	34.24	0.89
5-tap linear, 3-tap 3rd order Volterra EQ (Hybrid)	31.23	0.879

To improve the convergence one can use multi-step size in the LMS or use Kalman algorithm.

5 SNR Computation & Performance Comparison

To quantitatively compare the performance of various configurations of the equalizers, SNR (signal-to-noise) measure was developed. The SNR of the readback marks from the magneto-optic drive is defined as -

$$SNR = 10\log_{10}\left(\frac{W_{unit}^2}{\sigma^2}\right) \qquad (6)$$

where W_{unit} is the width of the minimum mark size recorded and

$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N} d_i^2$, $d_i = W_i - \widehat{W}_i$, where W_i & $\widehat{W}_i$ are the original and readback mark sizes.

Fig. 2 shows the SNR as a function of the minimum mark width of the raw marks, marks obtained with a linear 3-tap equalizer and marks obtained using a 3-tap 3rd order Volterra equalizer. From the graph it is very clear that the nonlinear equalization using Volterra yields an additional performance gain of about 1.3-1.5 dB over a linear equalizer and about 3 dB gain over un-equalized readback marks.

The SNR increases as the minimum mark width is increased because the relative amount of ISI decreases.

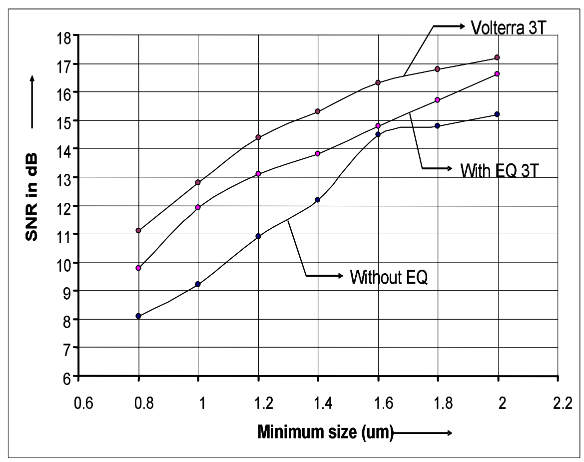

Fig. 2. SNR with and without Volterra Equalization.

Fig. 3 shows the SNR comparison of N-tap linear, N-tap 3rd order Volterra and Hybrid (N-tap linear + 3-tap 3rd order Volterra) equalization versus the # of taps. Using hybrid equalization one can obtain very good performance in signal quality at a modest increase in computational resources. Using pure Volterra does not payoff much as it reaches a point of extreme diminishing return very quickly, in this case after 5 taps. The gain in SNR going from linear to hybrid equalization is about 1.3 dB at 5-taps and beyond.

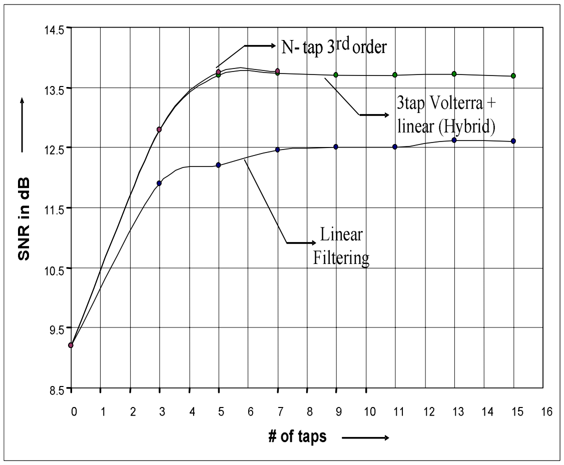

Fig. 3. SNR with Linear, pure Volterra and Hybrid Equalization.

6 Conclusion

The Volterra/Hybrid equalizers provided an extra 1.3-1.5 dB SNR gain over linear equalizers. Furthermore, it was observed that going to higher order Volterra equalizers (e.g. 5-tap 5th order or order) does not yield any extra benefit as one reaches a point of diminishing return immediately after 3-tap 3rd order.

Convergence issues related to equalizers were investigated. It was found that the eigenvalue spread of the covariance matrix was large for the Volterra equalizers compared to linear equalizers which resulted in somewhat slower convergence. This can be mitigated by employing a multiple step LMS algorithm.

The Hybrid equalizers provided the best trade-off between computational complexity and performance.

The extra gain in signal quality obtained can be utilized in various ways such as – achieving higher storage density, less expensive read/write optical assembly, lower BER and less stringent FEC (forward error correction) in the overall system design. The Volterra/Hybrid equalizer proposed and developed here can be applied to other communication systems (e.g. wireless, optical) exhibiting nonlinear ISI channel characteristics.

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